Numerical Tutorial: Continuous Gravitational Wave Computational Physics

Quantitative Problem Solving Guide

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Problem 1: Gravitational Wave Strain and Required Ellipticity

A targeted continuous wave search is monitoring a newly discovered millisecond pulsar located at a distance $d = 250$ pc. Electromagnetic observations show that the pulsar spins at a frequency $f_{\rm rot} = 400$ Hz. Assuming the star emits pure mass-quadrupole radiation at $f_{\rm gw} = 2f_{\rm rot}$ and possesses a canonical principal moment of inertia $I_{zz} = 10^{38}$ kg $\cdot$ m^2, calculate the minimum dimensionless ellipticity ϵ required for the star to be detectable by a terrestrial interferometer with a strain sensitivity threshold of $h_0 = 2.0 \times 10^{-26}$.
(Constants: $G = 6.674 \times 10^{-11}$ $m^3 kg^{-1} s^{-2}$, $c = 2.998 \times 10^8$ m/s, 1 $pc = 3.086 \times 10^{16}$ m)

Detailed Solution

The standard formula for the intrinsic gravitational wave strain amplitude h_0 from an asymmetric triaxial rotating neutron star is given by:

$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I_{zz} \epsilon f_{\rm gw}^2}{d} \tag{1}$$

First, we convert the distance d from parsecs to meters:

$$d = 250 \times (3.086 \times 10^{16} \text{ m}) = 7.715 \times 10^{18} \text{ m} \tag{2}$$

Identify the gravitational wave emission frequency for a standard orthogonal mass quadrupole:

$$f_{\rm gw} = 2 \times f_{\rm rot} = 2 \times 400 \text{ Hz} = 800 \text{ Hz} \tag{3}$$

Isolate the dimensionless ellipticity ϵ from the strain equation:

$$\epsilon = \frac{h_0 \cdot d \cdot c^4}{4\pi^2 \cdot G \cdot I_{zz} \cdot f_{\rm gw}^2} \tag{4}$$

Substitute the parameters into our rearranged expression:

$$\epsilon = \frac{(2.0 \times 10^{-26}) \times (7.715 \times 10^{18}) \times (2.998 \times 10^8)^4}{4\pi^2 \times (6.674 \times 10^{-11}) \times (10^{38}) \times (800)^2} \tag{5}$$

Evaluate the components of the numerator and denominator:

$$\text{Numerator} = (2.0 \times 10^{-26}) \times (7.715 \times 10^{18}) \times (8.0776 \times 10^{33}) \approx 1.2464 \times 10^{27} \tag{6}$$

$$\text{Denominator} = 4\pi^2 \times (6.674 \times 10^{-11}) \times 10^{38} \times 640000 \approx 1.6865 \times 10^{34} \tag{7}$$

$$\epsilon = \frac{1.2464 \times 10^{27}}{1.6865 \times 10^{34}} \approx 7.39 \times 10^{-8} \tag{8}$$

Answer: The minimum dimensionless ellipticity required is $\epsilon \approx 7.39 \times 10^{-8}$.

Problem 2: Gravitational Wave Contribution to Pulsar Spin-Down

A young energetic pulsar has a measured spin frequency $f = 30$ Hz and a first frequency derivative $\dot{f} = -3.0\times10^{-10}$ Hz/s. A second derivative measurement yields an observational braking index of exactly $n = 4.2$. Assuming the total rotational energy loss is driven exclusively by a combination of vacuum magnetic dipole radiation ($n = 3$) and mass-quadrupole continuous gravitational wave emission ($n = 5$), calculate the fraction of the total power currently being dissipated via continuous gravitational waves.

Detailed Solution

The total spin-down torque can be written phenomenologically as the sum of the magnetic dipole component and the gravitational wave component:

$$\dot{f} = \dot{f}_{\rm MD} + \dot{f}_{\rm GW} = -K_3 f^3 - K_5 f^5 \tag{9}$$

Differentiating this expression with respect to time via the chain rule yields:

$$\ddot{f} = \frac{d\dot{f}}{dt} = \left(-3K_3 f^2 - 5K_5 f^4\right)\dot{f} \tag{10}$$

The standard experimental definition of the braking index is $n = \frac{f\ddot{f}}{\dot{f}^2}$. Substituting our expression for $\ddot{f}$ gives:

$$n = \frac{f\left(-3K_3 f^2 - 5K_5 f^4\right)\dot{f}}{\dot{f}^2} = \frac{-3K_3 f^3 - 5K_5 f^5}{\dot{f}} \tag{11}$$

Since $\dot{f} = -K_3 f^3 - K_5 f^5$, we rearrange the numerator expansion:

$$n\dot{f} = 3(-K_3 f^3) + 5(-K_5 f^5) = 3\dot{f}_{\rm MD} + 5\dot{f}_{\rm GW} \tag{12}$$

We substitute $\dot{f}_{\rm MD} = \dot{f} - \dot{f}_{\rm GW}$ into the system to isolate the gravitational components:

$$n\dot{f} = 3(\dot{f} - \dot{f}_{\rm GW}) + 5\dot{f}_{\rm GW} = 3\dot{f} + 2\dot{f}_{\rm GW} \tag{13}$$

Isolate the gravitational wave spin-down contribution ratio ($\frac{\dot{f}_{\rm GW}}{\dot{f}}$):

$$2\dot{f}_{\rm GW} = (n-3)\dot{f} \implies \frac{\dot{f}_{\rm GW}}{\dot{f}} = \frac{n-3}{2} \tag{14}$$

Substitute the observationally measured braking index value ($n = 4.2$):

$$\frac{\dot{f}_{\rm GW}}{\dot{f}} = \frac{4.2-3}{2} = \frac{1.2}{2} = 0.60 \tag{15}$$

Because the total kinetic power dissipation scales linearly with the spin-down rate ($\frac{dE}{dt} = 4\pi^2 I_{zz} f\dot{f}$), the power loss fraction directly matches the frequency derivative loss fraction:

$$\text{Power Fraction}_{\rm GW} = \frac{\dot{f}_{\rm GW}}{\dot{f}} = 0.60 \tag{16}$$

Answer: Exactly 60% of the pulsar's total energy dissipation is driven by continuous gravitational wave emission.

Problem 3: Crustal Mountain Scale from Internal Strains

Suppose an asymmetric internal field magnetic configuration inside a neutron star exerts an uneven pressure, producing a localized density perturbation $\Delta\rho = 1.5 \times 10^{13}$ kg/m^3 over a crustal pocket volume $V = 4.0 \times 10^{10}$ m^3. This localized pocket sits at an average distance $r = 9.5$ km from the star's center, offset orthogonally from the rotation axis. Calculate the resulting dimensionless ellipticity ϵ of the star.
(Take the star's baseline mass as $M = 1.4M_\odot = 2.784 \times 10^{30}$ kg, radius $R = 11$ km, and model it as a uniform sphere where the principal moment of inertia is $I_{zz} = \frac{2}{5}MR^2$.)

Detailed Solution

The dimensionless ellipticity is mathematically defined as:

$$\epsilon = \frac{|I_{xx} - I_{yy}|}{I_{zz}} \tag{17}$$

The localized internal density perturbation creates an effective structural mass pocket asymmetry ΔM:

$$\Delta M = \Delta\rho \times V = (1.5 \times 10^{13} \text{ kg/m}^3) \times (4.0 \times 10^{10} \text{ m}^3) = 6.0 \times 10^{23} \text{ kg} \tag{18}$$

Treating this mass asymmetry as a point perturbation located at an orthogonal distance r from the longitudinal spin axis, the difference between the transverse moments of inertia is:

$$\begin{aligned} \Delta I = |I_{xx} - I_{yy}| &= \Delta M \times r^2 \\ &= (6.0 \times 10^{23} \text{ kg}) \times (9500 \text{ m})^2 = 5.415 \times 10^{31} \text{ kg} \cdot \text{m}^2 \end{aligned} \tag{19}$$

Next, calculate the canonical unperturbed background principal moment of inertia I_{zz} for the uniform sphere model:

$$\begin{aligned} I_{zz} = \frac{2}{5}MR^2 &= 0.4 \times (2.784 \times 10^{30} \text{ kg}) \times (11000 \text{ m})^2 \\ &= 1.3475 \times 10^{38} \text{ kg} \cdot \text{m}^2 \end{aligned} \tag{20}$$

Now, evaluate the final dimensionless ellipticity ratio:

$$\epsilon = \frac{\Delta I}{I_{zz}} = \frac{5.415 \times 10^{31} \text{ kg} \cdot \text{m}^2}{1.3475 \times 10^{38} \text{ kg} \cdot \text{m}^2} \approx 4.02 \times 10^{-7} \tag{21}$$

Answer: The resulting dimensionless ellipticity of the neutron star is $\epsilon \approx 4.02 \times 10^{-7}$.

Problem 4: X-Ray Flux Torque Balance Calculations

An accreting neutron star in a Low-Mass X-ray Binary (LMXB) has reached a stable rotational equilibrium spin frequency ceiling of $f_{\rm rot} = 600$ Hz due to torque balance. The observed bolometric X-ray flux from accretion is $F_X = 2.5 \times 10^{-11}$ W/m^2. Under the torque-balance hypothesis, the angular momentum added by the accreted matter is balanced entirely by the angular momentum radiated away via continuous gravitational waves. Calculate the expected gravitational wave strain amplitude h_0 received at Earth. *(Assume a canonical stellar radius $R = 10$ km and that matter accreted from the disk transfers momentum at the stellar radius).*

Detailed Solution

Under the torque-balance hypothesis, the angular accretion spin torque ($\tau_{\rm acc}$) balances the gravitational wave radiation braking torque ($\tau_{\rm gw}$):

$$\tau_{\rm acc} = \tau_{\rm gw} \tag{22}$$

The mass accretion accretion torque is driven by mass accretion rate $\dot{M}$ at the surface boundary:

$$\tau_{\rm acc} = \dot{M}\sqrt{GMR} \tag{23}$$

The total accretion luminosity is related to mass transfer by $L_X = \frac{GM\dot{M}}{R}$. Since the observed flux is $F_X = \frac{L_X}{4\pi d^2}$, the mass accretion rate is expressed as:

$$\dot{M} = \frac{4\pi d^2 F_X R}{GM} \implies \tau_{\rm acc} = 4\pi d^2 F_X R\sqrt{\frac{R}{GM}} \tag{24}$$

The torque exerted by gravitational wave emission matches the radiated power divided by angular frequency:

$$\tau_{\rm gw} = \frac{P_{\rm gw}}{\Omega_{\rm rot}} = \frac{c^3 d^2 h_0^2}{8\pi G f_{\rm rot}} \tag{25}$$

Equating the accretion torque to the radiation torque reveals that the distance variable (d^2) cancels out:

$$4\pi d^2 F_X R\sqrt{\frac{R}{GM}} = \frac{c^3 d^2 h_0^2}{8\pi G f_{\rm rot}} \implies h_0^2 = \frac{32\pi^2 G F_X R f_{\rm rot}}{c^3}\sqrt{\frac{R}{GM}} \tag{26}$$

Substitute the canonical constants for the neutron star ($M = 1.4M_\odot = 2.784 \times 10^{30}$ kg and $R = 10^4$ m):

$$\sqrt{\frac{R}{GM}} = \sqrt{\frac{10^4}{(6.674 \times 10^{-11}) \times (2.784 \times 10^{30})}} \approx 7.336 \times 10^{-9}\ \text{s} \tag{27}$$

Compute the final values for h_0^2:

$$\begin{aligned} h_0^2 &= \frac{32\pi^2 \times (6.674 \times 10^{-11}) \times (2.5 \times 10^{-11}) \times 10^4 \times 600 \times (7.336 \times 10^{-9})}{(2.998 \times 10^8)^3} \\ &= \frac{1.390 \times 10^{-17}}{2.694 \times 10^{25}} \approx 5.159 \times 10^{-43} \end{aligned} \tag{28}$$

$$h_0 = \sqrt{5.159 \times 10^{-43}} \approx 7.18 \times 10^{-22} \tag{29}$$

Answer: The expected continuous gravitational wave strain amplitude is $h_0 \approx 7.18 \times 10^{-22}$.

Problem 5: Black Hole Superradiance Particle Mass Constraints

An all-sky blind search for continuous gravitational waves targeting boson signals from superradiant clouds around isolated stellar-mass black holes detects a highly monochromatic continuous wave candidate at $f_{\text{gw}} = 483.6$ Hz. If this signal is driven by the coherent annihilation of ultralight axions ($a + a \to g$) populating a cloud around a black hole, calculate the rest mass m_a of the axion particle in electron-volts (eV).
(Constants: Planck's constant $h = 4.1357 \times 10^{-15}$ eV·s, speed of light $c = 2.998 \times 10^{8}$ m/s)

Detailed Solution

In black hole superradiant cloud physics, continuous gravitational wave emission driven by coherent pair annihilation converts the mass-energy of two bound scalar bosons into a single outward-propagating graviton:

$$E_{\text{graviton}} = hf_{\text{gw}} = 2m_a c^2 \tag{30}$$

We isolate the rest mass energy ($m_a c^2$) of the scalar particle directly in units of electron-volts (eV):

$$m_a c^2 = \frac{hf_{\text{gw}}}{2} \tag{31}$$

Substitute the detected signal frequency alongside the value for Planck's constant:

$$\begin{aligned} m_a c^2 &= \frac{(4.1357 \times 10^{-15} \text{ eV} \cdot \text{s}) \times 483.6 \text{ Hz}}{2} \\ &= \frac{2.0000 \times 10^{-12} \text{ eV}}{2} = 1.00 \times 10^{-12} \text{ eV} \end{aligned} \tag{32}$$

Answer: The rest mass of the underlying dark matter axion candidate is $m_a = 1.00 \times 10^{-12}$ **eV**.

2 Part 2: Different Types of CW Searches

Problem 6: Statistical False-Alarm Exceedance and $\mathcal{F}$-Statistic Thresholds

In a targeted coherent search using the $\mathcal{F}$-statistic under the null hypothesis (pure Gaussian noise without a signal), the statistical metric $2\mathcal{F}$ follows a central chi-squared (χ^2) distribution with 4 degrees of freedom. The survival function (probability that noise alone exceeds a threshold value $2\mathcal{F}_0$) is given by $P(2\mathcal{F} > 2\mathcal{F}_0) = \left(1 + \frac{2\mathcal{F}_0}{2}\right) e^{-2\mathcal{F}_0/2}$. Calculate the required $2\mathcal{F}_0$ value needed to limit the false-alarm probability to $\alpha = 1.0 \times 10^{-7}$ for a single template.

Detailed Solution

We construct the transcendental boundary equation mapping the noise survival function to the target single-bin false alarm rate limit α:

$$\left(1 + \frac{2\mathcal{F}_0}{2}\right) e^{-2\mathcal{F}_0/2} = 1.0 \times 10^{-7} \tag{33}$$

Let $x = \frac{2\mathcal{F}_0}{2} = \mathcal{F}_0$. This reduces the expression to:

$$(1 + x)e^{-x} = 1.0 \times 10^{-7} \tag{34}$$

Applying a natural logarithm transformations to both sides separates the polynomial and exponential terms:

$$\ln(1 + x) - x = \ln(10^{-7}) \approx -16.118 \implies x - \ln(1 + x) = 16.118 \tag{35}$$

We isolate x using numerical root-finding or manual iterative calculations:

- Evaluate $x = 18.00$: $18 - \ln(19) = 18 - 2.944 = 15.056$ (Too low)
- Evaluate $x = 19.50$: $19.5 - \ln(20.5) = 19.5 - 3.020 = 16.480$ (Too high)
- Evaluate $x = 19.12$: $19.12 - \ln(20.12) = 19.12 - 3.002 = 16.118$ (Exact convergence)

Since $x = \mathcal{F}_0 = 19.12$, we recover the structural search metric parameter $2\mathcal{F}_0$:

$$2\mathcal{F}_0 = 2 \times 19.12 = 38.24 \tag{36}$$

Answer: The required metric threshold value for detection is $2\mathcal{F}_0 = 38.24$.

Problem 7: Doppler Shift Bin Dragging in the Stack-Slide Pipeline

A Stack-Slide semi-coherent search pipeline uses Short Fourier Transforms (SFTs) of length $T_{\text{sft}} = 1800$ s to search for signals near a target frequency $f_0 = 500$ Hz. Calculate the width of a single frequency bin in the SFT. Next, determine the maximum number of frequency bins a continuous wave signal could drift across over a 6-month period due to Earth's orbital velocity around the Sun.
(Assume Earth moves in a circular orbit with a velocity $v_{earth} \approx 30$ km/s).

Detailed Solution

The intrinsic frequency bin width resolution (Δf_{bin}) of an un-padded Short Fourier Transform is the inverse of its time duration:

$$\Delta f_{\text{bin}} = \frac{1}{T_{\text{sft}}} = \frac{1}{1800 \text{ s}} \approx 5.556 \times 10^{-4} \text{ Hz} \tag{37}$$

The maximum amplitude of the annual orbital Doppler modulation occurs when the line of sight to the continuous wave source lies directly within Earth's orbital plane. The peak single-sided frequency shift is modeled as:

$$\Delta f_{\text{Doppler}} = f_0 \times \frac{v_{\text{earth}}}{c} = 500 \text{ Hz} \times \frac{30000 \text{ m/s}}{2.998 \times 10^8 \text{ m/s}} \approx 0.05003 \text{ Hz} \tag{38}$$

As Earth moves from one side of the Sun to the exact opposite side over a 6-month baseline, the total peak-to-peak frequency drift range corresponds to twice the single-sided Doppler modulation value:

$$\text{Total Shift Range} = 2 \times \Delta f_{\text{Doppler}} = 2 \times 0.05003 \text{ Hz} = 0.10006 \text{ Hz} \tag{39}$$

To calculate the total number of individual SFT frequency channels the drifting signal sweeps through, divide the total modulation range by the discrete bin width:

$$\text{Number of Bins} = \frac{\text{Total Shift Range}}{\Delta f_{\text{bin}}} = \frac{0.10006 \text{ Hz}}{5.556 \times 10^{-4} \text{ Hz}} \approx 180.1 \tag{40}$$

Answer: The discrete SFT bin width is 5.56×10^{-4} **Hz**, and the signal drifts across a maximum of 180 **bins** over 6 months.

Problem 8: Parameter Metric Resolution and Grid Spacing

In a wide-parameter all-sky continuous wave search, the spacing of templates in the frequency domain must satisfy a maximum mismatch constraint. According to the phase information metric tensor, the frequency component of the metric is $g_{ff} = \frac{\pi^2 T_{\text{coh}}^2}{3}$. If a pipeline limits the maximum allowed mismatch to $m_{\text{max}} = 0.11$ and uses a coherence time $T_{\text{coh}} = 80$ hours, calculate the maximum allowed grid spacing step (Δf) in the template bank.

Detailed Solution

The fractional signal-to-noise loss mismatch (m) resulting from an offset step size (Δf) between grid templates is defined via differential geometry as:

$$m = g_{ff}(\Delta f)^2 \tag{41}$$

To bound the maximum potential template mismatch to the limit m_{max}, we equate the metric function to our boundary constraint:

$$m_{\text{max}} = \frac{\pi^2 T_{\text{coh}}^2}{3}(\Delta f)^2 \tag{42}$$

Convert the coherence processing time block window (T_{coh}) from hours to standard metric seconds:

$$T_{\text{coh}} = 80 \times 3600 \text{ s} = 2.88 \times 10^5 \text{ s} \tag{43}$$

Isolate the frequency grid resolution step parameters (Δf):

$$(\Delta f)^2 = \frac{3 \cdot m_{\text{max}}}{\pi^2 \cdot T_{\text{coh}}^2} \implies \Delta f = \frac{1}{\pi T_{\text{coh}}}\sqrt{3 m_{\text{max}}} \tag{44}$$

Substitute the values into our isolated grid equation:

$$\begin{aligned} \Delta f &= \frac{1}{\pi \times (2.88 \times 10^5)}\sqrt{3 \times 0.11} \\ &= (1.1052 \times 10^{-6}) \times 0.57446 \approx 6.35 \times 10^{-7} \text{ Hz} \end{aligned} \tag{45}$$

Answer: The maximum allowed frequency grid spacing step is $\Delta f \approx 6.35 \times 10^{-7}$ **Hz**.

Problem 9: Astrophysical Bounds from LVK O3 Upper Limits

An LVK all-sky continuous wave search over the O3 dataset establishes an upper-limit strain sensitivity boundary of $h_0^{95\%} = 1.2 \times 10^{-25}$ at an emission frequency $f_{\rm gw} = 200$ Hz. If there is an undiscovered, fast-spinning isolated neutron star located within our galaxy at a distance $d = 1.5$ kpc, calculate the maximum dimensionless ellipticity ϵ this star could have without being detected by this search.
(Take the principal moment of inertia to be the canonical value $I_{zz} = 10^{38}$ kg $\cdot$ m^2).

Detailed Solution

The equation governing mass-quadrupole continuous gravitational wave strain limits is:

$$h_0^{95\%} = \frac{4\pi^2 G}{c^4} \frac{I_{zz}\epsilon f_{\rm gw}^2}{d} \tag{46}$$

Convert the astrophysical distance parameters from kiloparsecs to meters:

$$d = 1.5 \times 1000 \times (3.086 \times 10^{16} \text{ m}) = 4.629 \times 10^{19} \text{ m} \tag{47}$$

Rearrange the equation to isolate the maximum allowable structural ellipticity limit variable ϵ:

$$\epsilon = \frac{h_0^{95\%} \cdot d \cdot c^4}{4\pi^2 \cdot G \cdot I_{zz} \cdot f_{\rm gw}^2} \tag{48}$$

Substitute the constants alongside the observational limits established during the O3 search run:

$$\epsilon = \frac{(1.2 \times 10^{-25}) \times (4.629 \times 10^{19}) \times (2.998 \times 10^8)^4}{4\pi^2 \times (6.674 \times 10^{-11}) \times (10^{38}) \times (200)^2} \tag{49}$$

Evaluate the numerical products within both the numerator and denominator parameters:

$$\text{Numerator} = (1.2 \times 10^{-25}) \times (4.629 \times 10^{19}) \times (8.0776 \times 10^{33}) \approx 4.487 \times 10^{28} \tag{50}$$

$$\text{Denominator} = 4\pi^2 \times (6.674 \times 10^{-11}) \times 10^{38} \times 40000 \approx 1.054 \times 10^{32} \tag{51}$$

$$\epsilon = \frac{4.487 \times 10^{28}}{1.054 \times 10^{32}} \approx 4.26 \times 10^{-4} \tag{52}$$

Answer: The maximum allowable structural dimensionless ellipticity for such a star is $\epsilon \approx 4.26 \times 10^{-4}$.

Problem 10: Coherent vs. Semi-Coherent Sensitivity Loss Scaling

A data analysis pipeline processes a continuous wave dataset spanning a total observation time $T_{\text{obs}} = 120$ days. The pipeline can run either a fully coherent search or a semi-coherent search. The semi-coherent configuration divides the dataset into segments of length $T_{\text{coh}} = 2$ days and sums the accumulated power across segments. Calculate the ratio of the minimum detectable gravitational wave strain amplitude for the fully coherent search ($h_{0,\text{min}}^{\text{coh}}$) to that of the semi-coherent configuration ($h_{0,\text{min}}^{\text{semi}}$) for a fixed target statistical false-alarm threshold.

Detailed Solution

The strain amplitude sensitivity threshold for a fully coherent matched-filtering pipeline follows an analytical scaling laws governed by the inverse square root of total observation time:

$$h_{0,\text{min}}^{\text{coh}} \propto \frac{1}{\sqrt{T_{\text{obs}}}} \tag{53}$$

For a semi-coherent stack-sliding style search layout that partitions data into N segments of tracking length T_{coh} (where $N = T_{\text{obs}}/T_{\text{coh}}$), the statistical noise averaging changes the sensitivity scaling rule to the inverse fourth root:

$$h_{0,\text{min}}^{\text{semi}} \propto \frac{1}{(T_{\text{obs}} \cdot T_{\text{coh}})^{1/4}} \tag{54}$$

Dividing the coherent threshold expression by the semi-coherent equivalent creates a simplified dimensional ratio:

$$\frac{h_{0,\text{min}}^{\text{coh}}}{h_{0,\text{min}}^{\text{semi}}} = \frac{\frac{1}{\sqrt{T_{\text{obs}}}}}{\frac{1}{(T_{\text{obs}} \cdot T_{\text{coh}})^{1/4}}} = \left(\frac{T_{\text{obs}} \cdot T_{\text{coh}}}{T_{\text{obs}}^2}\right)^{1/4} = \left(\frac{T_{\text{coh}}}{T_{\text{obs}}}\right)^{1/4} \tag{55}$$

Substitute the timeline parameters ($T_{\text{coh}} = 2$ days and $T_{\text{obs}} = 120$ days) directly into the equation:

$$\frac{h_{0,\text{min}}^{\text{coh}}}{h_{0,\text{min}}^{\text{semi}}} = \left(\frac{2 \text{ days}}{120 \text{ days}}\right)^{1/4} = \left(\frac{1}{60}\right)^{1/4} \approx 0.359 \tag{56}$$

Answer: The ratio of the minimum detectable strain amplitudes is ≈ 0.359.